SECTION 3.1 SPIRALS[1] (1995)

3.1.1 PURPOSE (1995)

a. A spiral or transition curve should be used in main-line tracks, if practicable, between tangent and curve or between the different degrees of curvature of a compound curve. A spiral is also desirable in all tracks other than main tracks, where practicable, between tangent and curves and between the different degrees of curvature of a compound curve. The form of the spiral should be such that the degree of curvature increases directly with the length. In other than main tracks with no superelevation, this permits the engine or car trucks to be directed gradually to their rotated position (required by a curve) rather than to be directed instantaneously. This will likewise tend to prevent distortion of the alignment of the main body of the curve due to the force required to produce angular acceleration. In main tracks with superelevation, this permits a uniform rate of change of elevation of the outer rail on the spiral and promotes best riding conditions by maintaining throughout the train passage from tangent to curve the desired relation between amount of superelevation and degree of curvature.

b. The desirable length of the spiral for tracks other than main tracks with no superelevation is the maximum possible consistent with economy and the purpose of the track layout.

c. The desirable length of the spiral for main tracks where the alignment is being entirely reconstructed or where the cost of the realignment of the existing track will not be excessive should be such that when passenger cars of average roll tendency are to be operated the rate of change of the unbalanced lateral acceleration acting on a passenger will not exceed 0.03 *g* per sec Equation (1). Also, the desirable length in this case needed to limit the possible racking and torsional forces produced should be such that the longitudinal slope of the outer rail with respect to the inner rail will not exceed $1/744$ Equation (2), which is based on an 85-foot long car.

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1 References, Vol. 12, 1911, part 1, pp. 417, 462; Vol. 16, 1915, pp. 731, 1145; Vol. 37, 1936, pp. 466, 1018; Vol. 41, 1940, pp. 602, 867; Vol. 42, 1941, pp. 636, 836; Vol. 48, 1947, pp. 553, 885; Vol. 54, 1953, pp. 972, 1398; Vol. 63, 1962, pp. 487, 753; Vol. 66, 1965, pp. 501, 763. Reapproved with revisions 1995.

(1) The formulas recommended to obtain the above results are:

$$L = 1.63\ (E_u)\ V \qquad \textbf{EQ 1}$$

where:

$L(min)$ = desirable length of spiral in feet
E_u = unbalanced elevation in inches
V = maximum train speed in miles per hour

NOTE: If the spiral is to be designed for passenger equipment which has the car-body roll with respect to the track controlled by special designs, the length of spiral for this specific roll angle may be determined by the method and formula given on pages 94, 516 and 517, Vol. 65, AREMA Proceedings.

$$L = 62\ E_a \qquad \textbf{EQ 2}$$

where:

$L(min)$ = desirable length of spiral in feet
E_a = actual elevation in inches

(2) In using the above formulas for determining the length of spirals joining tangents and curves and joining curves of different radii, the maximum length of spiral produced by the two formulas should be used.

d. It is recognized that in the case of realignment of existing tracks, EQ 1 may produce a length of spiral the construction of which would result in excessive costs. Therefore, in such cases it is felt that the length should be such that, with average roll tendency of passenger cars operated on the track, the rate of change of the unbalanced lateral acceleration acting on a passenger will not exceed 0.04 g per sec Equation (3). In this case the maximum slope EQ 2 should be retained.

(1) The formula recommended for this case in lieu of formula EQ 1 is:

$$L = 1.22\ E_u\ V \qquad \textbf{EQ 3}$$

where:

$L(min)$ = desirable length of spiral in feet
E_u = unbalanced elevation in inches
V = maximum train speed in miles per hour

(2) In using EQ 2 and EQ 3 for determining the length of spirals joining tangents and curves and joining curves of different radii, the maximum length of spiral produced by the two formulas should be used.

3.1.2 THE SPIRAL CURVE (1965)

The following formulas, using the notation given, are recommended for location of the spiral curve. These formulas are based upon the fundamental relation that the degree d of the spiral at any point increases in constant relation to the lengths along the spiral in stations or $d = ks$. The term k represents the rate at which the degree of curvature increases, and its value should be selected so the spiral will attain the degree of curvature of the circular curve in a length not less than given by EQ 1, EQ 2 or EQ 3.

3.1.3 NOTATION (1965)

a. In designations for curve points, the first initial represents the alignment on the ride towards station zero, the second that away from station zero.

b. Figure 5-3-1 is a diagram illustrating the application of spirals at each end of a circular curve with the stationing from the left. The notation used in the formulas will be evident from this diagram and from the following:

D = degree of circular curve
d = degree of curvature of the spiral at any point
l = Length from the T.S. or S.T., to any point on the spiral having coordinates x and y
s = length l in 100-foot stations
L = total length of spiral
S = length L in 100-foot stations
δ = central angle of the spiral from the T.S. or S.T. to any point on the spiral
Δ = central angle of the whole spiral
a = deflection angle from the tangent at the T.S. or S.T. to any point on the spiral
b = orientation angle from the tangent at any point on the spiral to the T.S. or S.T.
k = increase in degree of curvature per 100-foot station along the spiral

c. All functions are in feet or degrees unless otherwise noted.

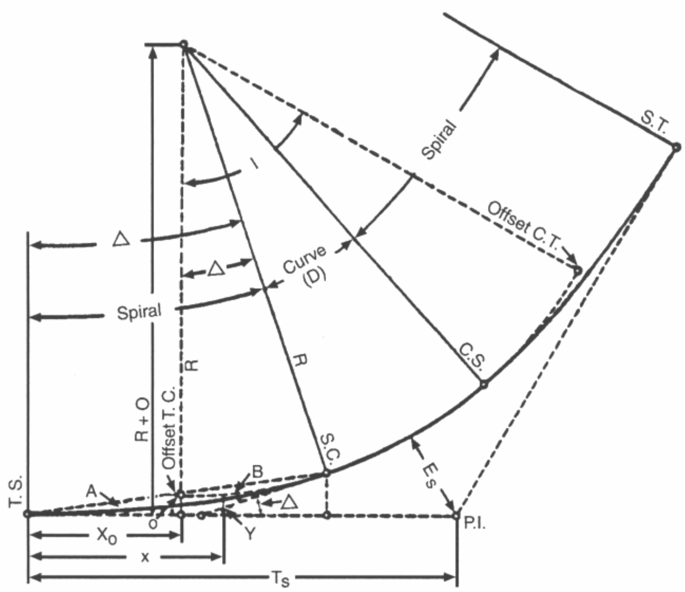

Figure 5-3-1. Spiral Applications

3.1.4 FORMULAS (2005)[1]

$$d = ks = \frac{kl}{100};\ D = kS = \frac{kL}{100} \quad \text{EQ 4}$$

$$\delta = \frac{1}{2}ks^2 = \frac{dl}{200};\ \Delta = \frac{1}{2}kS^2 = \frac{DL}{200} \quad \text{EQ 5}$$

$$a = \frac{1}{3}\delta = \frac{1}{6}ks^2;\ A = \frac{1}{3}\Delta = \frac{1}{6}kS^2 \quad \text{EQ 6}$$

$$b = \frac{2}{3}\delta;\ B = \frac{2}{3}\Delta \quad \text{EQ 7}$$

$$y = 0.582\delta s - 0.00001264\delta^3 s \quad \text{EQ 8}$$

$$x = l - 0.003048\delta^2 s \quad \text{EQ 9}$$

$$o = 0.1454\Delta S \quad \text{EQ 10}$$

$$Xo = \frac{1}{2}L - 0.000508\Delta^2 S \quad \text{EQ 11}$$

$$T_s = (R + o)\tan\left(\frac{I}{2}\right) + Xo \quad \text{EQ 12}$$

$$E_s = (R + o)\operatorname{exsec}\left(\frac{I}{2}\right) + o \quad \text{EQ 13}$$

3.1.5 STAKING SPIRALS BY DEFLECTIONS (1965)

a. From EQ 10, EQ 11 and EQ 12, the T.S. and S.T. may be located from the P.I. of the curve Figure 5-3-1. EQ 13 is useful in adjusting the degree D of the circular curve if it is desired to limit the throw of the center of the curve, or balance the throw of existing track.

b. The entire spiral may then be run from the T.S. or S.T., using EQ 6 to determine the deflection angle a from the tangent to any point on the spiral.

c. Deflection angles with the transit at any point on the spiral other than the T.S. may be determined from the principle that the spiral at the transit point deflects from a circular curve having the same degree as the spiral at that point at the same rate as it does from the tangent at the T.S. To continue the spiral from any intermediate transit point, the transit is backsighted on the T.S. with an angle set off equal to twice the deflection angle from the T.S. to the transit point. The transit will then read zero along the tangent to the spiral at that point. For any succeeding spiral point, the deflection angle for a circular

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[1] The use of computer alignment software, even in chord definition mode, may generate results for curve length, stationing and tangent offsets that deviate from the values calculated using the equation in Article 3.1.4. Regardless of the software package used, it is recommended that the alignment designer evaluate and confirm the acceptability of the results of any computer alignment output with the individual railroad or transit property's design policy.

curve having the same degree as the spiral at the transit point and a length equal to the distance from the transit to the spiral point is then calculated; to this is added the deflection angle for the same length of spiral but calculated as it would be from the T.S.

d. To locate the spiral with the transit at the S.C. or C.S., the deflection angles to set points on the spiral are equal to the deflection angles for the corresponding points on the circular curve (extended) less the deflection angles of the spiral from the circular curve. The deflection angles of the spiral from the circular curve are the same as for the corresponding lengths of the spiral from the T.S.

e. In staking by deflection, it is sometimes convenient to divide the spiral into a number of equal chords. The first or initial deflection a_1 may be calculated for the first chord point. The deflections for the following chord points are a_1 times the chord number squared.

3.1.6 STAKING SPIRALS BY OFFSETS (1965)

The spiral may be staked to the midpoint by right-angle offsets from the tangent and from there to the S.C. by normal offsets from the circular curve (between the offset T.C. and the S.C.). The offset at midpoint equals ½ o and the other offsets vary as the cubes of the distances from the T.S. or the S.C.

3.1.7 APPLYING THE SPIRAL TO COMPOUND CURVES (1965)

a. In applying a spiral between two circular curves of a compound curve, the length of spiral is determined from the speed of operation and the difference in elevation of the two circular curves (EQ 1, EQ 2, or EQ 3). The spiral offset o may be found from the formula given using a value of D equal to the difference in the degrees of curvature of the two circular curves. The spiral extends for one-half its length each side of the offset point of compound curvature. The spiral deflects from the inside of the flatter curve and from the outside of the sharper curve at the same rate as it would from the tangent. The spiral may be staked by deflection angles from either end. If the transit is located at the spiral point on the flatter curve, reading zero when sighting along the tangent to the circular curve, the deflection angles to set points on the spiral are equal to the deflection angles for corresponding points on the circular curve (extended) *plus* the deflection angles of the spiral. If the transit is set at the spiral point on the sharper curve, the deflection angles are equal to the deflection angles for that circular curve (extended) *minus* the deflection angles for the spiral.

b. As an alternative, the spiral can be staked out by offsets from the two circular curves. The offset at the middle point of the spiral equals ½ o, and the other offsets vary as the cubes of the distances from the ends of the spiral.

SECTION 3.2 STRING LINING OF CURVES BY THE CHORD METHOD[1] (2006)

3.2.1 SCOPE (2006)

a. String lining of curves may be used to supplement the engineer's survey or the alignment system of track maintenance equipment. The method outlined below is applicable to both circular and spiral curves where the angle between the tangents does not change. This is an iterative method that will enable a trained user to develop the throws at each station and smooth the horizontal alignment throughout the curve. Briefly, the method consists of dividing the curve to be lined into 31-foot stations,

[1] References, Vol. 36, 1935, pp. 561, 977; Vol. 41, 1940, pp. 557, 867; Vol. 54, 1953, pp. 973, 1398; Vol. 63, 1962, pp. 487, 753. For more detailed information see Vol. 34, 1933, pp. 493–508. Reapproved with revisions 1962.

recording the mid-ordinates of the chords spanning each two stations, and designating a reasonable amount of throw to each station.

b. The purpose of string lining is to obtain a curve that is smooth and offers good ride quality. This result can be obtained by developing an alignment in which the mid-ordinates at each station of the circular curve are as nearly uniform as possible. A considerable difference in the mid-ordinates of the circular part of the curve should be avoided.

c. String lining is based on the following:

(1) The mid-ordinates of a circular curve are indicative of its degree of curvature. Therefore, the mid-ordinates of a circular curve of uniform radius are equal for a chord of uniform length. The mid-ordinates of a spiral curve will vary incrementally along the length of the spiral.

(2) For all practical purposes for curves with more than 193-foot radius (less than 30 degrees of curvature) the mid-ordinate of a given length chord varies directly with the degree of curve. The sum of the mid-ordinates of the realigned curve must equal the sum of the mid-ordinates of the original curve.

(3) The throw at any station on a curve will change the ordinate at that point equal to the throw.

(4) The throw will increase or decrease the mid-ordinate at adjacent stations by an amount equal to one-half the throw – always increasing when the throw decreases and decreasing when the throw increases the mid-ordinate.

3.2.2 TOOLS REQUIRED (1965)

A strong fish line or chord of 62-ft length and a 50-ft steel tape; marking crayon; a suitable rule graduated to inches and tenths thereof with the graduations beginning at the extreme end of the rule or scale; and a pad of forms (described later).

3.2.3 PROCESS (1965)

a. All work is done on the outside rail of the curve. First stand on tangent several rail lengths back from the curve and locate the beginning of the curve as closely as possible by eye. This point is Station 0 and should be so marked.

b. The station 31 ft back along the tangent is Station –1.

c. Beginning at Station –1, lay off with steel tape and mark each 31-foot point and number consecutively as Stations –1, 0, 1, 2, 3, etc., and continue the stationing at least two stations beyond the point of tangent, which is also located by eye. These station numbers are entered in Col. 1 of the sample form shown in Table 5-3-1.

d. Beginning at Station 0, measure mid-ordinates in tenths of inches from the outside rail to the line joining Stations –1 and 1. This is entered in Col. 2 of the same form. Proceed around the curve to the P.T., measuring the mid-ordinate at each station and entering on the form.

e. Take track centers at frequent intervals where there is more than one track and record any obstacle which might affect lining, noting same in Col. 9 of the sample form shown in Table 5-3-1.

f. Column descriptions:

(1) Col. 1 is for station numbers.

(2) Col. 2 is for measured mid-ordinates in inches and tenths thereof.

(3) Col. 3 is for revised mid-ordinates.

(4) Col. 4, headed Difference, shows the difference between the ordinates in Cols. 2 and 3.

(5) Cols. 5 and 6 are explained by the headings.

(6) Col. 7 is for the full throw which is double the figure shown in Col. 6.

Negative throw indicates that the track at that station is to be thrown in, while positive throw indicates that the track is to be thrown out.

(7) Col. 8 is obtained by subtracting algebraically the full throw from one-half the gage, 28.25″.

(8) Col. 9 is also for revised superelevation to be used on the curve, and, of course, is contingent upon the maximum speed of trains running over this curve.

g. By inspection of measured ordinates in Col. 2, the beginning and ending of the spiral curves can be located as nearly correct as possible.

h. In the example in Table 5-3-1 the end of the east spiral is taken at Station 7, while the end of the west spiral is taken at Station 24.

i. In Col. 4 are entered the differences between the measured ordinate, Col. 2, and the revised ordinate, Col. 3, in tenths of an inch. If the ordinate in Col. 3 at any station is larger than that in Col. 2, the sign of the difference in Col. 4 is minus. Conversely, if the revised ordinate is less than the measured ordinate the sign of the difference is plus.

j. In Col. 5 are entered the algebraic sums of the differences (shown in Col. 4) up to and including stations being entered. The operation is performed in sequence as indicated by arrows.

k. In Col. 6 the half-throw is entered. The result shown here is the algebraic sum of Col. 5 up to and including the preceding station. The operation is also performed in the order shown by the arrows.

l. In the example, computations based on selected revised spiral ordinates in subcolumn A Table 5-3-1, carried through to Col. 6, indicate a half-throw of 31 or a full-throw of 6.2 inches at Station 7, which is too great. The minus sign of the half-throw indicates that ordinates slightly smaller should be selected. Slightly smaller spiral ordinates are therefore entered in subcolumn B and the curve ordinate of 45 is carried out through Station 11. This gives too great a throw in the positive direction.

m. Therefore, interpolate a spiral between those used in subcolumns A and B, and enter these new spiral ordinates in subcolumn C. The curve ordinate of 46 is carried out a few stations below the S.C. (spiral curve) at Station 7. Computing a third time through to column 6, the half-throw at Station 7 is –26 and at Station 11 is –10 which gives a practical throw. For trial the circular curve ordinate of 46 in Col. 3 is carried through to Station 23 (one station back on the circular curve from the C.S. (curve spiral) and extension made to Col. 6, where the half-throw is +31.

n. Sum up in Col. 2 the measured ordinates from Stations 24 to 32, incl., in the original spiral, which total 190. The sum of the measured ordinates from Stations 0 to 23, incl., is 922, bringing the total sum of measured ordinates to 1112. The sum of the revised ordinates from Stations 0 to 23, incl., is 921. To assure that Col. 5 will end in 0, the sum of the revised spiral ordinates from stations 24 to 32, incl., must equal 191. Such revised spiral ordinates are entered in subcolumn "C."

o. Carrying the calculations through to Col. 6, the sum of differences check out 0, but the final half-throw is + 17, indicating that the trial spiral ends in a parallel tangent. To end in the original tangent, both Cols.

Table 5-3-1. Sample Form – Stationing from East to West

	Station Numbers	Ordinates					Difference				Sum of Differences Up to and Including the Station				Half Throw Sum Col. 5 up to and Including Preceding Station				Full Throw	Gage to Tack	Remarks
		Meas-ured	Revised				Col.2 – Col.3														
	1	2	3				4				5				6				7	8	9
			A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D			
	–1	0	0	0	0		0	0	0		0	0	0		0	0	0		0	28.25	
T.S.	0	1	1	1	1		0	0	0		0	0	0		0	0	0		0	28.25	
	1	5	7	6	7		–2	–1	–2		–2	–1	–2		0	0	0		0	28.25	
	2	14	13	13	13		+1	+1	+1		–1	0	–1		–2	–1	–2		–4	28.65	
	3	16	20	19	20		–4	–3	–4		–5	–3	–5		–3	–1	–3		–6	28.85	
	4	26	27	26	26		–1	0	0		–6	–3	–5		–8	–4	–8		–1.6	29.85	
	5	30	34	32	33		–4	–2	–3		–10	–5	–8		–14	–7	–13		–2.6	30.35	
	6	43	40	39	40		+3	+4	+3		–7	–1	–5		–24	–12	–21		–4.2	32.45	
S.C.	7	56	46	44	45		+10	+12	+11		+3	+11	+6		–31	–13	–26		–5.2	33.45	
	8	49	47	45	46		+2	+4	+3		+5	+15	+9			–2	–20		–4.0	32.25	
	9	35	47	45	46		–12	–10	–11		–7	+5	–2			+13	–11		–2.2	30.45	
	10	51	47	45	46		+4	+6	+5		–3	+11	+3			+18	–13		–2.6	30.85	
	11	49	47	45	46		+2	+4	+3		–1	+15	+6			+29	–10		–2.0	30.25	
	12	43			46				–3				+3			+44	–4		–.8	29.05	
	13	45			46				–1				+2				–1		–.2	28.45	
	14	50			46	47			+4	+3			+6	+5			+1	+1	+.2	28.05	
	15	49			46				+3				+9	+8			+7	+6	+1.2	27.05	
	16	43			46				–3				+6	+5			+16	+14	+2.8	25.45	
	17	38			46				–8				–2	–3			+22	+19	+3.8	24.45	
	18	50			46				+4				+2	+1			+20	+16	+3.2	25.05	
	19	55			46				+9				+11	+10			+22	+17	+3.4	24.85	
	20	33			46				–13				–2	–3			+33	+27	+5.4	22.85	
	21	44			46				–2				–4	–5			+31	+24	+4.8	23.45	
	22	50			46	47			+4	+3			0	–2			+27	+19	+3.8	24.45	
	23	47			46				+1				+1	–1			+27	+17	+3.4	24.85	
C.S.	24	48			47	46			+1	+2			+2	+1			+28	+16	+3.2	25.05	
	25	38			41				–3				–1	–2			+30	+17	+3.4	24.85	
	26	37			34				+3				+2	+1			+29	+15	+3.0	25.25	
	27	21			27				–6				–4	–5			+31	+16	+3.2	25.05	
	28	18			20				–2				–6	–7			+27	+11	+2.2	26.05	
	29	17			14	13			+3	+4			–3	–3			+21	+4	+.8	27.45	
	30	9			7				+2				–1	–1			+18	+1	+2	28.05	
S.T.	31	2			1				+1				0	0			+17	0	0	28.25	
	32	0			0				0				0	0			+17	0	0	28.25	
	Sum	1112				1112															

5 and 6 must balance; therefore, an adjustment of the revised ordinates is necessary and is made according to the following rule.

p. When the final half-throw is positive, subtract from the revised ordinates having high station numbers and add an equal amount to the ordinates having low station numbers, choosing stations in pairs such that the sum of the differences of station numbers, taken in pairs, equals the numerical amount of the final half-throw. When the final half-throw is negative, reverse the procedure, subtracting from the ordinates having low station numbers and adding to those having high station numbers.

q. Since in Table 5-3-1 the final half-throw is +17, an ordinate (or ordinates) of a low station number will have to be increased and that of a high station number decreased. As it is desirable to keep the spiral uniform, let us change Station 24 from 47 to 46 and Station 22 from 46 to 47. This change will decrease the final half-throw by 2 or 1 × (Sta 24 – Sta 22). Let us now change Station 29 from ordinate 14 to ordinate 13. Then following the rule, subtract (17 – 2 = 15) from Station 29, leaving 14 and increase the ordinate at Station 14 from 46 to 47. Enter these revised ordinates in subcolumn "D," carrying out these computations again to Col. 6, the final half-throw becomes 0 and the ordinates are balanced.

r. Computations are simplified by treating the entries in Cols. 2, 3, 4, 5 and 6 as whole numbers, and placing decimal points in Col. 7, as shown in Table 5-3-1.

s. In working out string lining problems considerable assistance can be gained by platting the measured mid-ordinates against the station numbers. Figure 5-3-2 shows the results for the curve given in Table 5-3-1. By platting the mid-ordinates in this manner, the ends of the spiral, as well as points of compounding, can be determined readily and an estimate of the average ordinate to use on the circular curve section can be closely determined.

t. When tabulations are completed and the curve staked, a copy of the form should be given the track foreman to enable him to apply the proper superelevation at the various stations as the track is lined.

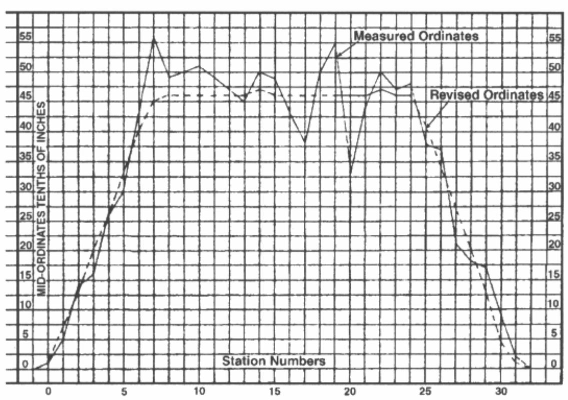


Figure 5-3-2. Platting Mid-ordinates

c. Some critical conditions might be observed in any yard. For example a simple No. 8 turnout to a track paralleling the one from which it diverges on 13-foot centers will provide approximately 41 ft of tangent between the toe of frog and point of reverse curve, which is probably not sufficient for critical consists between nearly 12 degree curves. A No. 8 turnout to parallel track on 14-foot center increases the tangent to about 49 feet which should cause no problems.

d. For curves above 13 degrees, the maximum coupler angle is exceeded regardless of the length of tangent between curves. Curves above 13 degrees should, therefore, be avoided.

e. Extreme long car-short car combinations should not be operated- over reverse curves of 10 degrees or larger.

3.5.2 WITH SPIRALS AND SUPERELEVATION (1984)

a. The minimum tangent length between reverse curves with spirals and superelevation should not be less than the length of the longest car that is to traverse the curves.

b. Consideration should also be given to the chord length being used by the automatic lining equipment when establishing the minimum tangent length.

SECTION 3.6 VERTICAL CURVES (2002)

a. Vertical curves as calculated in item (f) below should be used to connect all changes in gradients.

b. The length of vertical curve is determined by changes in gradient, vertical acceleration and the speed of the train.

c. The purpose of the vertical curve is to ease the change of the gradients in order to reduce coupler and diaphragm binding and eliminate the danger of breaking trains in two as a direct result of train action. In addition, the proper vertical curve will provide for passenger comfort on passenger trains. Vertical curves should be designed as long as physically and econmically possible.

d. A vertical curve which is concave upwards shall be denoted as a sag. A vertical curve which is concave downwards shall be denoted as a summit.

e. The vertical curve may be either circular or parabolic in shape.

f. The **minimum** length of the vertical curve for both sags and summits is determined by the following formula (except that in no case should the length of the vertical curve be less than 100 feet long):

$$L = \frac{D \times V^2 \times K}{A}$$

Where: A = vertical acceleration in feet/sec/sec (ft/sec^2)

D = Absolute value of the difference in rates of grades expressed as a decimal

K = 2.15 conversion factor to give L in feet

L = Length of vertical curve in feet

V = Speed of the train in miles per hour

g. The recommended vertical acceleration (A) should be selected based on the type of operations and is the same for both sags and summits.

Freight Operations:

A = 0.10 feet/sec/sec

Passenger and Transit Operations:

A = 0.60 feet/sec/sec

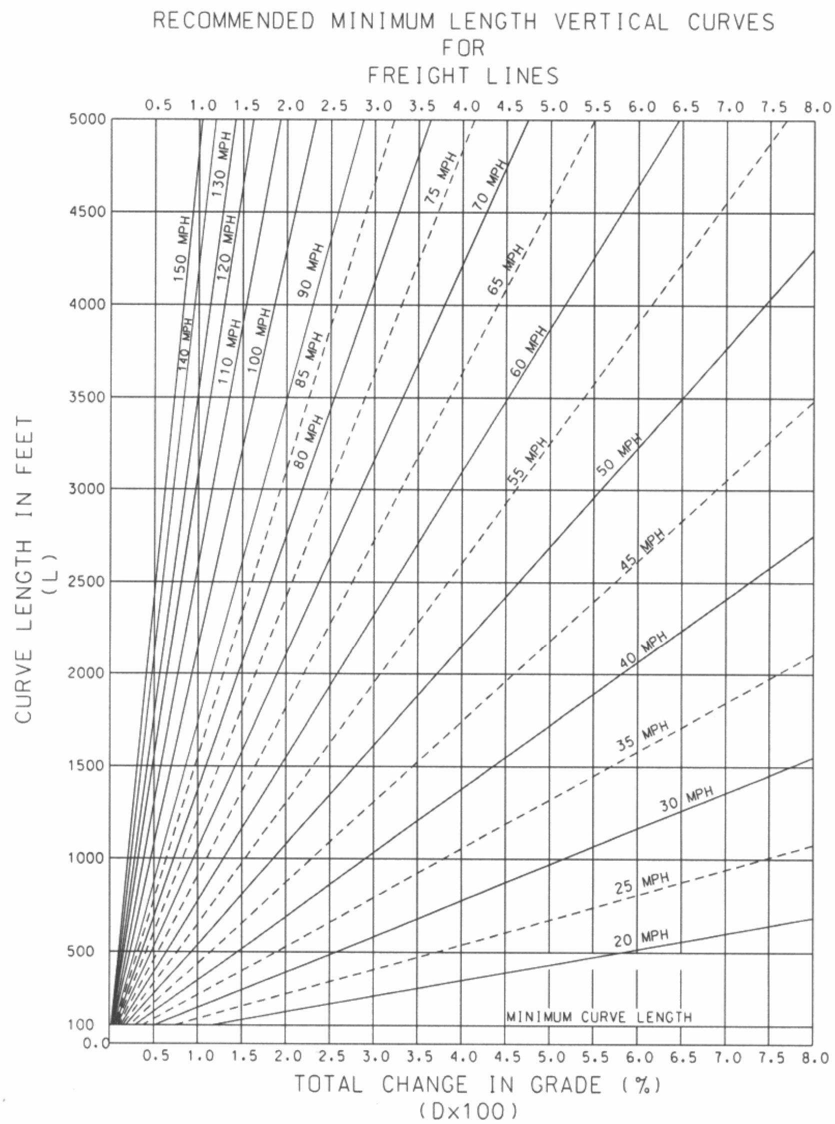
h. The minimum distance between vertical curves shall not be less than 100 ft.

i. The train speed which should be used in the above formula for establishing the length of vertical curve should be the maximum speed found on that particular subdivision or route. Special attention should be paid to locations where local conditions have dictated a speed restriction now in place, but where such a restriction might be removed at a later date.

j. It is not recommended to place turnouts within the limits of a vertical curve.

k. Curves constructed to this formula should not present any problems for the current generation of equipment. Slow speed curves, such as hump crests, should, however, be designed with consideration for vertical clearance rather than using this formula.

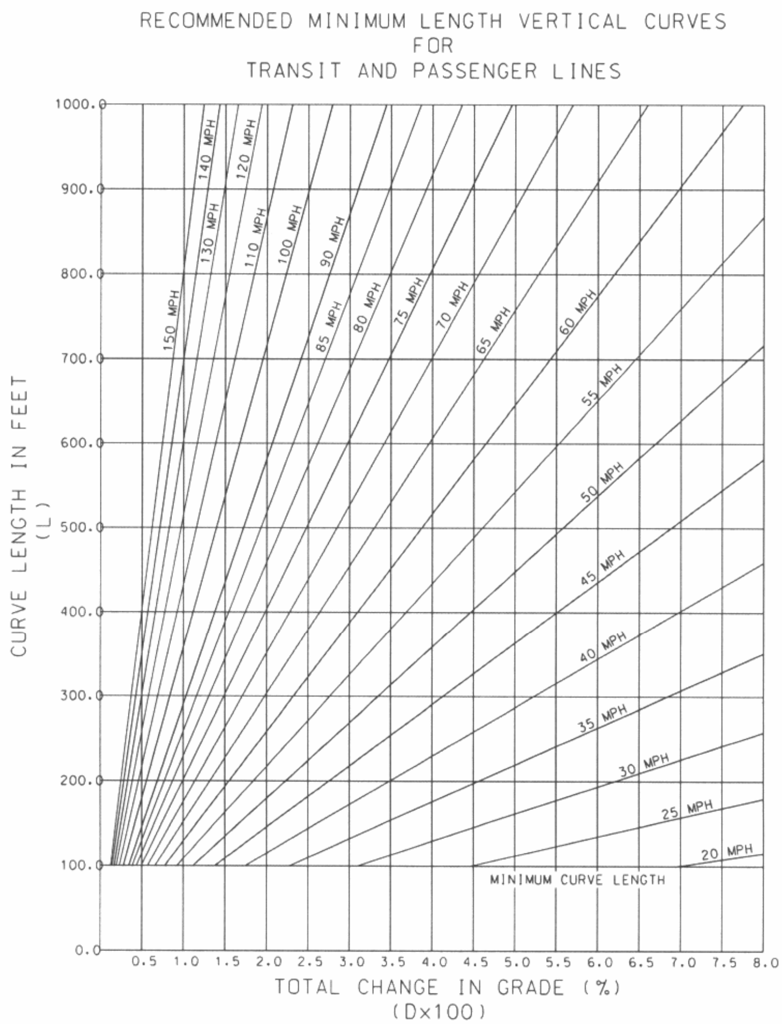
NOTE: Values for various speeds and change in gradients have been graphed for reference.



FORMULA: $L = (D \times V^2 \times K)/A$

WHERE: V = Speed of the train in miles per hour.
L = Length of vertical curve in feet.
D = Absolute value of the difference in rates of grades expressed as a decimal.
A = 0.1 feet/sec/sec.
K = 2.15 = conversion factor to give L in feet.

Figure 5-3-3. Recommended Minimum Length Vertical Curves for Freight Lines

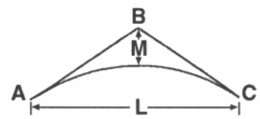


FORMULA: $L = (D \times V^2 \times K)/A$

WHERE: V = Speed of the train in miles per hour.
L = Length of vertical curve in feet.
D = Absolute value of the difference in rates of grades expressed as a decimal.
A = 0.6 feet/sec/sec.
K = 2.15 = conversion factor to give L in feet.

Figure 5-3-4. Recommended Minimum Length Vertical Curves for Transit and Passenger Lines

l. One such form of vertical curve is developed as follows:



L = Length of vertical curve in 100-ft stations

M = Correction in elevation at B

When vertical curve is concave downwards –

$$M = \frac{(\text{Elev B} \times 2) - (\text{Elev A} + \text{Elev B})}{4}$$

When vertical curve is concave upwards –

$$M = \frac{(\text{Elev A} + \text{Elev C}) - (\text{Elev B} \times 2)}{4}$$

m. The correction for any other point on a vertical curve is proportional to the square of its distance from A or C to B.

n. Corrections are – when the vertical curve is concave downwards and + when the vertical curve is concave upwards

Example Calculation for Freight Operations

Crest curve with 0.50% ascending grade meeting a 0.50% descending grade. Maximum design speed is 50 MPH.

A = 0.10 feet/sec/sec vertical acceleration (Freight)

D = Absolute value of ((+.005) - (-.005)) = 0.01

K = 2.15 conversion factor to give L in feet

V = 50 MPH design speed

$$L = \frac{D \times V^2 \times K}{A} = \text{Length of vertical curve in feet}$$

$$L = \frac{(0.01) \times (50\text{MPH})^2 \times 2.15}{0.10 \text{ feet/sec/sec}} = 537.50 \text{ feet} \quad \text{say } 540 \text{ feet}$$

Example Calculation for Passenger and Transit Operations

Sag curve with 0.50% descending grade meeting a 0.50% ascending grade. Maxiumum design speed is 75 MPH.

A = 0.60 feet/sec/sec vertical acceleration (Passenger and Transit)
D = Absolute value of ((-.005) - (+.005)) = 0.01
K = 2.15 conversion factor to give L in feet
V = 75 MPH design speed

$$L = \frac{D \times V^2 \times K}{A} = \text{Length of vertical curve in feet}$$

$$L = \frac{(0.01) \times (75\text{MPH})^2 \times 2.15}{0.60\ \text{feet/sec/sec}} = 201.56 \text{ feet} \quad \text{say } 205 \text{ feet}$$

SECTION 3.7 COMPENSATED GRADIENTS (1999)

3.7.1 PROPOSED AREMA STANDARDS FOR COMPENSATED GRADIENTS (1999)

a. Compensation of gradients due to horizontal curvature is recommended on all gradients, but is essential on ruling gradients.

b. The purpose of the compensated gradient is to equate the total resistance of a train on a horizontal curve on a gradient to that of the total resistance of a train on tangent track on a gradient.

c. The amount of gradient compensation is determined by the compensation factor and the degree of curve.

d. The recommended compensation factor to be used is 0.04 percent per degree of curve. This corresponds to the resistance created by standard three piece trucks on non-lubricated curves.

e. The recommended compensated gradient due to curvature shall be calculated as follows:

$$G_c = G - 0.04D$$

Where: G = gradient before compensation, expressed in percent
D = degree of curve expressed in decimals of degrees
G_c = compensated gradient expressed in percent